

1 次のを計算せよ。

(1) $(1 + \sqrt{2} - \sqrt{3})^2$

(3) $\frac{1}{2 + \sqrt{5}} + \frac{\sqrt{5} - 3}{\sqrt{5} + 1}$

$$\begin{aligned} (1) & \{ (1 + \sqrt{2}) - \sqrt{3} \}^2 \\ &= (1 + \sqrt{2})^2 - 2(1 + \sqrt{2}) \times \sqrt{3} + \sqrt{3}^2 \\ &= 1 + 2\sqrt{2} + 2 - 2\sqrt{3} - 2\sqrt{6} + 3 \\ &= \underline{6 + 2\sqrt{2} - 2\sqrt{3} - 2\sqrt{6}} \end{aligned}$$

$$\begin{aligned} (3) & \frac{1}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}} + \frac{\sqrt{5} - 3}{\sqrt{5} + 1} \times \frac{\sqrt{5} - 1}{\sqrt{5} - 1} \\ &= \frac{2 - \sqrt{5}}{4 - 5} + \frac{5 - 4\sqrt{5} + 3}{5 - 1} \\ &= -(2 - \sqrt{5}) + \frac{8 - 4\sqrt{5}}{4} \\ &= -2 + \sqrt{5} + 2 - \sqrt{5} \\ &= \underline{0} \end{aligned}$$

(2) $(1 + \sqrt{2} - \sqrt{6})(1 - \sqrt{2} + \sqrt{6})$

(4) $\frac{\sqrt{7}}{\sqrt{7} - \sqrt{5}} - \frac{\sqrt{5}}{\sqrt{7} + \sqrt{5}}$

$$\begin{aligned} (2) & \{ 1 + (\sqrt{2} - \sqrt{6}) \} \{ 1 - (\sqrt{2} - \sqrt{6}) \} \\ &= 1^2 - (\sqrt{2} - \sqrt{6})^2 \\ &= 1 - (2 - 2 \cdot 2 \cdot \sqrt{3} + 6) \\ &= \underline{-7 + 4\sqrt{3}} \end{aligned}$$

$$\begin{aligned} (4) & \frac{\sqrt{7}(\sqrt{7} + \sqrt{5}) - \sqrt{5}(\sqrt{7} - \sqrt{5})}{(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5})} \\ &= \frac{7 + \sqrt{35} - \sqrt{35} + 5}{7 - 5} \\ &= \frac{12}{2} \\ &= \underline{6} \end{aligned}$$

$$\begin{aligned} & (x+y)^3 - 3xy(x+y) \\ &= 2\sqrt{3} - 3 \cdot 2 \cdot 2\sqrt{3} \\ &= 2\sqrt{3} - 12\sqrt{3} = \underline{-10\sqrt{3}} \end{aligned}$$

2 $x = \frac{2}{\sqrt{3} + 1}, y = \frac{2}{\sqrt{3} - 1}$ のとき、次の式の値を求めよ。

(1) $x + y$

(2) xy

(3) $x^2y + xy^2$

(4) $x^2 + y^2$

(5) $x^3 + y^3$

$$\begin{aligned} x &= \frac{2}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \\ &= \frac{2(\sqrt{3} - 1)}{3 - 1} \end{aligned}$$

$$\begin{aligned} y &= \frac{2}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\ &= \frac{2(\sqrt{3} + 1)}{3 - 1} \end{aligned}$$

(3), (4), (5) は
(1), (2) の
基本対称式
を利用する!!

$$x = \sqrt{3} - 1$$

$$y = \sqrt{3} + 1$$

(1) $x + y = \sqrt{3} - 1 + \sqrt{3} + 1 = \underline{2\sqrt{3}}$

(2) $xy = (\sqrt{3} - 1)(\sqrt{3} + 1) = 3 - 1 = \underline{2}$

(3) $x^2y + xy^2 = xy(x + y) = 2 \times 2\sqrt{3} = \underline{4\sqrt{3}}$

(4) $x^2 + y^2 = (x + y)^2 - 2xy = (2\sqrt{3})^2 - 2 \cdot 2 = 12 - 4 = \underline{8}$

(5) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$= (x + y)(x^2 + y^2 - xy)$

$= 2\sqrt{3} \times (8 - 2) = \underline{12\sqrt{3}}$

(4) は
利用する!!

3) $\frac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}+\sqrt{3}}$ の分母を有理化せよ。

$$\begin{aligned} & \frac{1+\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}} \times \frac{1+\sqrt{3}-\sqrt{2}}{1+\sqrt{3}-\sqrt{2}} = \frac{3+\sqrt{3}-\sqrt{2}-\sqrt{6}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} \\ & = \frac{(1+\sqrt{3})^2 - 2(1+\sqrt{3})\sqrt{2} + 2}{(1+\sqrt{3})^2 - 2} = \frac{3+\sqrt{3}-\sqrt{2}-\sqrt{6}-3\sqrt{3}-3+\sqrt{6}+3\sqrt{2}}{1-3} \\ & = \frac{1+2\sqrt{3}+3-2\sqrt{2}-2\sqrt{6}+2}{-2} = \frac{6+2\sqrt{3}-2\sqrt{2}-2\sqrt{6}}{2+2\sqrt{3}} \\ & = \frac{1+2\sqrt{3}+3-2}{2+2\sqrt{3}} = \frac{6+2\sqrt{3}-2\sqrt{2}-2\sqrt{6}}{2+2\sqrt{3}} \\ & = \frac{1}{2+2\sqrt{3}} \end{aligned}$$

3分の有理化は、
2回やる!!

4) $\frac{1}{2-\sqrt{3}}$ の整数部分を a, 小数部分を b とする。次の式の値を求めよ。

(1) a

$$\frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

$1 < \sqrt{3} < 2$
 $3 < 2+\sqrt{3} < 4$

(2) b

(1) $a = 3$

(2) $b = 2+\sqrt{3}-3 = \sqrt{3}-1$

(3) $a+2b+b^2+1$
 $= a+(b+1)^2$
 $= 3 + \{(\sqrt{3}-1)+1\}^2$
 $= 3 + \sqrt{3}^2 = 6$

因数分解
12から
代入!!

5) $\sqrt{2} = 1.4142, \sqrt{5} = 2.2361$ とするとき、次の値を求めよ。

(1) $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2.2361}{5} = 0.44722$

(2) $\frac{2+\sqrt{2}}{2(1+\sqrt{2})} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{2+\sqrt{2}-2\sqrt{2}-2}{2(1-2)} = \frac{-\sqrt{2}}{-2} = \frac{\sqrt{2}}{2} = \frac{1.4142}{2} = 0.7071$

(3) $\frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2} = \sqrt{5}+\sqrt{2} = 2.2361+1.4142 = 3.6503$

<今日のふりかえり>

