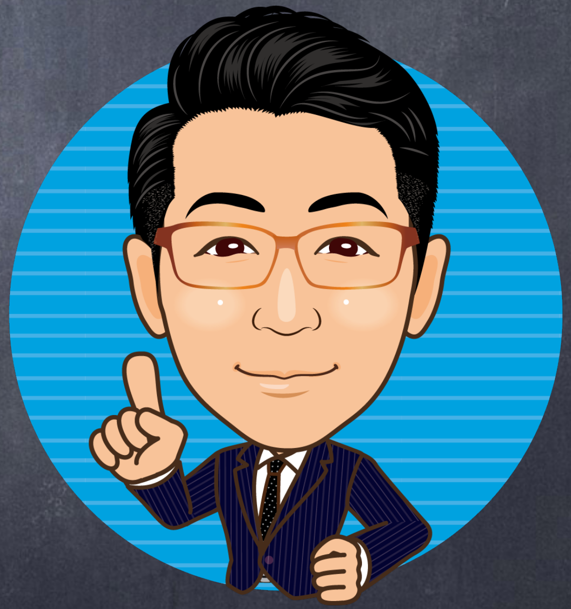
隣接３項間漸化式

教科書 p.102,103



例) $a_1=1,\ a_2=4,\ a_{n+2}-5a_{n+1}+6a_n=0$

<方針>

等比型へ変形

$$\alpha^2-5\alpha+6=0$$

$$(\alpha-2)(\alpha-3)=0$$

$$\alpha=2,\ 3$$

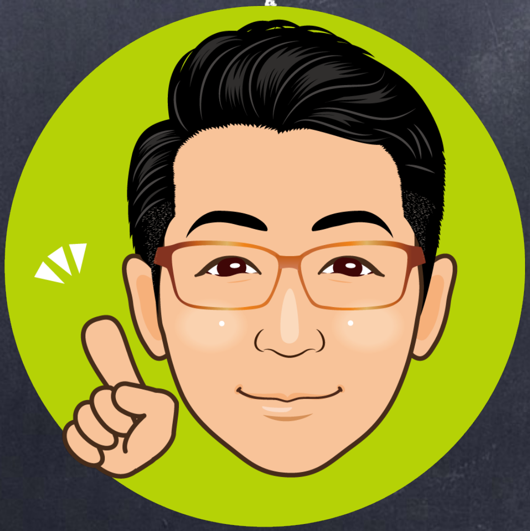
(与式) ⇔

$$a_{n+2}-3a_{n+1}=2(a_{n+1}-3a_n) \quad \cdots ①$$

と

$$a_{n+2}-2a_{n+1}=3(a_{n+1}-2a_n) \quad \cdots ②$$

に変形することができる!!



①について、

$$a_{n+2}-3a_{n+1}=2(a_{n+1}-3a_n)$$

$a_{n+1}-3a_n=b_n$ とおくと、

$a_{n+2}-3a_{n+1}=b_{n+1}$ となり。

$b_1=a_2-3a_1$

$=1$

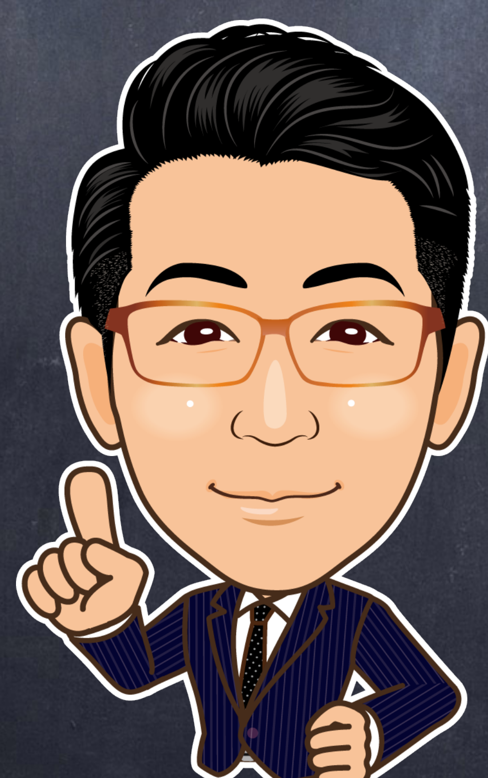
すなわち

①

$\iff b_{n+1}=2b_n$

$b_n=1\times 2^{n-1}$

$b_n=2^{n-1}$



つまり、

$$a_{n+1}-3a_n=2^{n-1}\quad\cdots①'$$

②について、

$$a_{n+2} - 2a_{n+1} = 3(a_{n+1} - 2a_n)$$

$a_{n+1} - 2a_n = c_n$ とおくと、

$a_{n+2} - 2a_{n+1} = c_{n+1}$ となり

$$c_1 = a_2 - 2a_1$$

$$= 2$$

すなわち

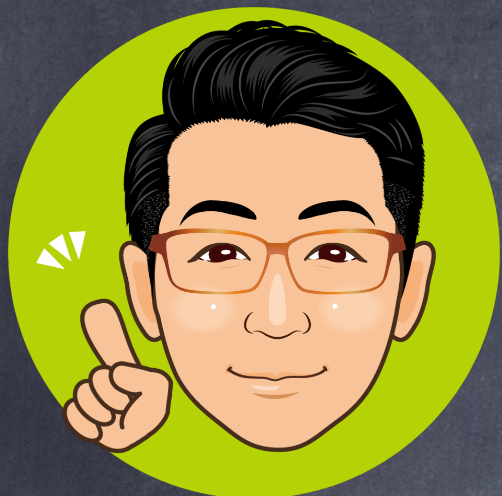
②

$\iff c_{n+1} = 3c_n$

$$c_n = 2\times 3^{n-1}$$

つまり

$$a_{n+1} - 2a_n = 2\cdot 3^{n-1} \cdots ②'$$



$$a_{n+1} - 3a_n = 2^{n-1} \quad \cdots ①$$

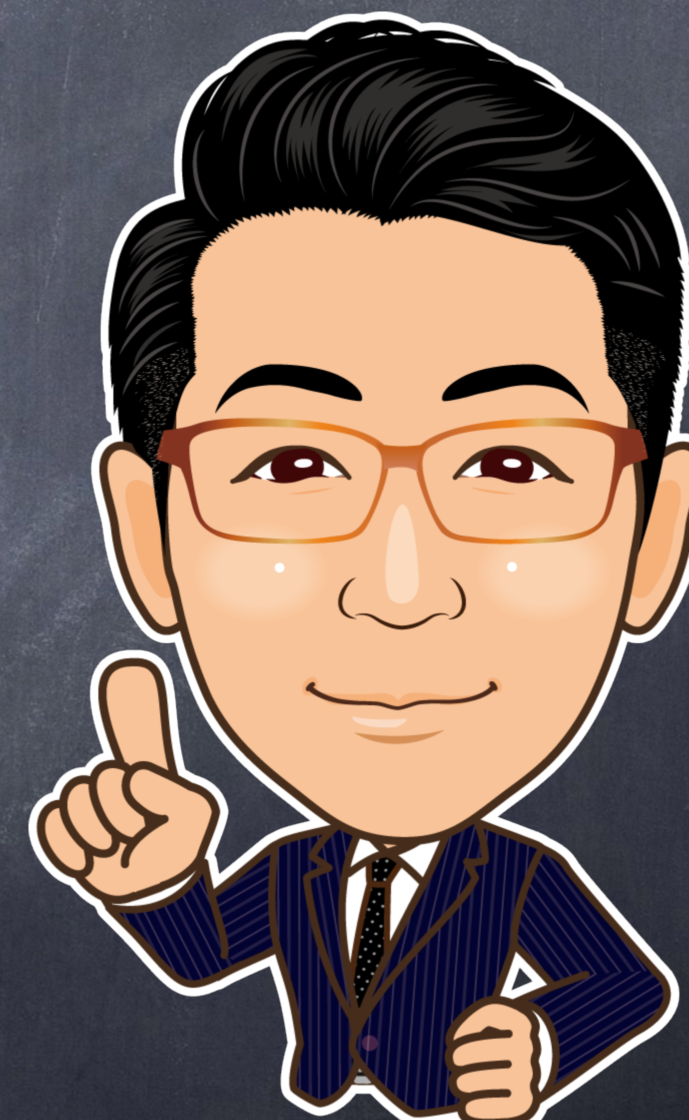
$$a_{n+1} - 2a_n = 2\cdot 3^{n-1} \quad \cdots ②$$

より

①－② より

$$-a_n = 2^{n-1} - 2\cdot 3^{n-1}$$

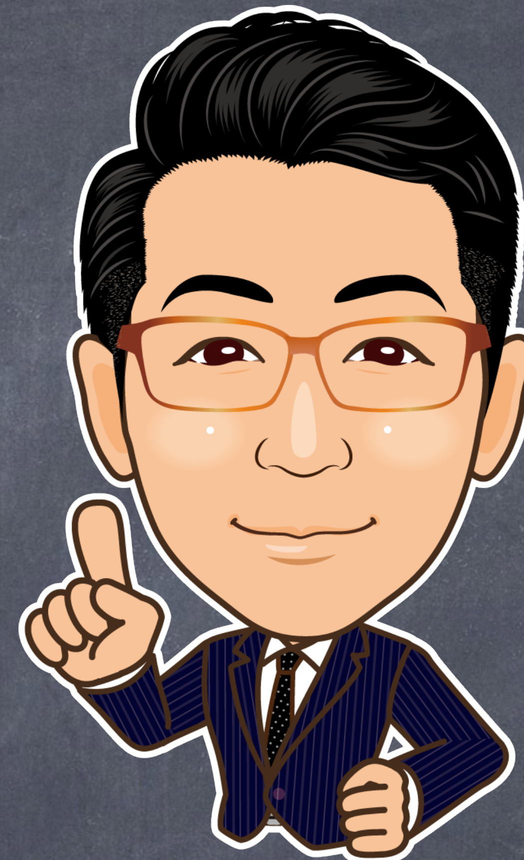
$$a_n = 2\cdot 3^{n-1} - 2^{n-1}$$



〈まとめ〉

$$a_{n+2}+pa_{n+1}+qa_n=0$$

$$\left(\begin{array}{c}\alpha^2+p\alpha+q=0\\ \alpha=r,\ s\end{array}\right)$$



$$a_{n+2}-ra_{n+1}=s(a_{n+1}-ra_n)$$

$\Longleftrightarrow$ または、

$$a_{n+2}-sa_{n+1}=r(a_{n+1}-sa_n)$$

と変形できる!!