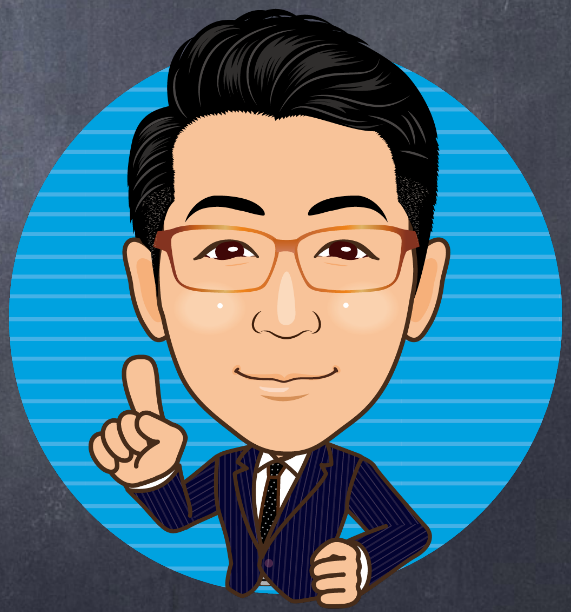
シグマ記号応用②

教科書 p.89,90



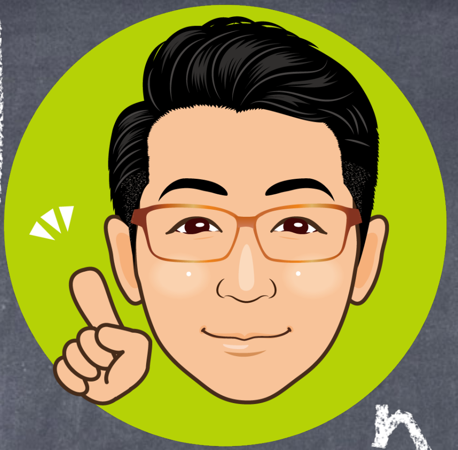
＜シグマの性質＞

① $$\sum_{k=1}^{n}(a_k+b_k)=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$$

② $$\sum_{k=1}^{n}pa_k=p\sum_{k=1}^{n}a_k$$

（pはkに無関係な数）

(ex) $\displaystyle\sum_{k=1}^{n}(k^2-3k+2)$ を求めよ.



$$\sum_{k=1}^{n}(k^2-3k+2)=\sum_{k=1}^{n}k^2-3\sum_{k=1}^{n}k+\sum_{k=1}^{n}2$$

それぞれ公式を代入!!

$$=\frac{1}{6}n(n+1)(2n+1)-3\times\frac{1}{2}n(n+1)+2n$$

$$=\frac{1}{6}n\{(n+1)(2n+1)-9(n+1)+12\}$$

$$=\frac{1}{6}n(2n^2-6n+4)=\frac{1}{3}n(n-1)(n-2)$$

すぐ展開
or
因数分解

(例) $1\cdot 3+2\cdot 4+\cdots+n(n+2)$ を求めよ.

<方針>

$\boxed{\Sigma\text{化} \Rightarrow \text{公式代入!!}}$

$$(\text{与式}) = \sum_{k=1}^{n} k(k+2) = \sum_{k=1}^{n} (k^2+2k)$$

$$= \sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$$

$$= \frac{1}{6}n(n+1)(2n+1) + 2\cdot\frac{1}{2}n(n+1)$$

$$= \underline{\underline{\frac{1}{6}n(n+1)(2n+7)}}$$