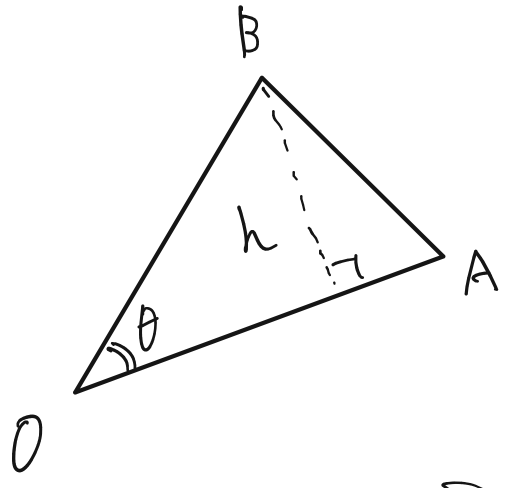
1　三角形の面積についてまとめよ。



① $S = \frac{1}{2} OA \cdot h$

$h = OB \sin\theta$

② $S = \frac{1}{2} OA \cdot OB \cdot \sin\theta$

③ $S = \frac{1}{2} |\overrightarrow{OA}| \cdot |\overrightarrow{OB}| \sin\theta$

$\cos\theta = \frac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$,　$0° \leqq \theta \leqq 180°$,　$\sin\theta \geqq 0$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1 - \frac{(\overrightarrow{OA} \cdot \overrightarrow{OB})^2}{|\overrightarrow{OA}|^2 |\overrightarrow{OB}|^2}}$

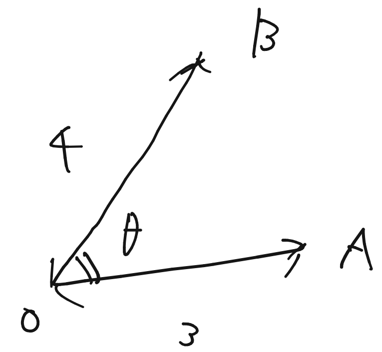
$S = \frac{1}{2} |\overrightarrow{OA}| \cdot |\overrightarrow{OB}| \times \sqrt{1 - \frac{(\overrightarrow{OA} \cdot \overrightarrow{OB})^2}{|\overrightarrow{OA}|^2 |\overrightarrow{OB}|^2}}$

④ $S = \frac{1}{2}\sqrt{|\overrightarrow{OA}|^2 |\overrightarrow{OB}|^2 - (\overrightarrow{OA} \cdot \overrightarrow{OB})^2}$

$\overrightarrow{OA} = (a_1, a_2)$,　$\overrightarrow{OB} = (b_1, b_2)$

⑤ $S = \frac{1}{2} |a_1 b_2 - a_2 b_1|$

2　$|\overrightarrow{OA}|=3$，$|\overrightarrow{OB}|=4$，$\overrightarrow{OA}\cdot\overrightarrow{OB}=6$ を満たす△OAB の面積 S を求めよ。



(解1)

$\cos\theta = \frac{6}{3 \times 4} = \frac{1}{2}$

$\theta = 60°$

$S = \frac{1}{2} \cdot 3 \times 4 \times \sin 60° = 3\sqrt{3}$

$S = 3\sqrt{3}$

(解2)

$S = \frac{1}{2}\sqrt{|\overrightarrow{OA}|^2 \cdot |\overrightarrow{OB}|^2 - (\overrightarrow{OA} \cdot \overrightarrow{OB})^2}$

$= \frac{1}{2}\sqrt{3^2 \times 4^2 - 6^2} = 3\sqrt{3}$

$S = 3\sqrt{3}$

③ 3点 O (0, 0), A (3, −1), B (2, 2) を頂点とする三角形の面積 S を求めよ。

(解1) $\overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$　$\angle AOB = \theta$ とすると

$\overrightarrow{OA} \cdot \overrightarrow{OB} = 6 - 2 = 4$

$|\overrightarrow{OA}| = \sqrt{9+1} = \sqrt{10}$

$|\overrightarrow{OB}| = \sqrt{4+4} = 2\sqrt{2}$

$\cos\theta = \dfrac{4}{\sqrt{10} \times 2\sqrt{2}} = \dfrac{1}{\sqrt{5}}$

$0° \leqq \theta \leqq 180°$ であるから $\sin\theta \geqq 0$

$\sin\theta = \sqrt{1 - \frac{1}{5}} = \dfrac{2}{\sqrt{5}}$

$S = \dfrac{1}{2} \cdot \sqrt{10} \times 2\sqrt{2} \times \dfrac{2}{\sqrt{5}} = 4$

$S = 4$

(解2)

$\overrightarrow{OA} \cdot \overrightarrow{OB} = 4$, $|\overrightarrow{OA}| = \sqrt{10}$, $|\overrightarrow{OB}| = 2\sqrt{2}$

$S = \dfrac{1}{2}\sqrt{|\overrightarrow{OA}|^2 |\overrightarrow{OB}|^2 - (\overrightarrow{OA} \cdot \overrightarrow{OB})^2} = \dfrac{1}{2}\sqrt{10 \times 8 - 16}$

$= \dfrac{1}{2} \times 8 = 4$

$S = 4$

(解3)

$S = \dfrac{1}{2}\left| 3 \times 2 - (-1) \times 2 \right|$

$= \dfrac{1}{2}|6+2| = \dfrac{1}{2} \cdot 8 = 4$　　$S = 4$

④ 3点 A $(1+\sqrt{3}, 2)$, B $(\sqrt{3}, 5)$, C $(4+\sqrt{3}, 1)$ を頂点とする三角形の面積 S を求めよ。

$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

基準とするベクトルを上手に設定しよう!!

$\overrightarrow{AB} \cdot \overrightarrow{AC} = -3 - 3 = -6$

$|\overrightarrow{AB}| = \sqrt{1+9} = \sqrt{10}$

$|\overrightarrow{AC}| = \sqrt{9+1} = \sqrt{10}$

$S = \dfrac{1}{2}\sqrt{(\sqrt{10})^2 \times (\sqrt{10})^2 - (-6)^2}$

$= \dfrac{1}{2}\sqrt{100 - 36}$

$= \dfrac{1}{2} \times 8 = 4$　　$S = 4$

③と同様に、(解1) や (解3) でも解けます!!

<今日のふりかえり>