1 次の関数を微分せよ。

(1) $y=(3x^2-x+1)^3$　　(2) $y=\dfrac{1}{(2x+1)^2}$　　(3) $y=\left(2+\dfrac{1}{x}\right)^4$

(1) $y'=3(3x^2-x+1)^2\times(3x^2-x+1)'$

$y'=3(3x^2-x+1)^2(6x-1)$

(3) $y'=4\left(2+\dfrac{1}{x}\right)^3\times\left(2+\dfrac{1}{x}\right)'$

$=4\left(2+\dfrac{1}{x}\right)^3\times(x^{-1})'$

$=4\left(2+\dfrac{1}{x}\right)^3\times(-1)\cdot x^{-2}$

(2) $y=(2x+1)^{-2}$

$y'=-2(2x+1)^{-3}\times(2x+1)'$

$y'=\dfrac{-4}{(2x+1)^3}$

$y'=-4\left(2+\dfrac{1}{x}\right)^3\times\dfrac{1}{x^2}$

2 次の関数を微分せよ。

(1) $y=x^{\frac{2}{5}}$　　(2) $y=\sqrt[6]{x^5}$　　(3) $y=\sqrt{x^2+4}$　　(4) $y=\dfrac{1}{\sqrt{3x+1}}$

(1) $y'=\dfrac{2}{5}x^{-\frac{3}{5}}$

$=\dfrac{2}{5}\cdot\dfrac{1}{\sqrt[5]{x^3}}$

(2) $y=x^{\frac{5}{6}}$

$y'=\dfrac{5}{6}x^{-\frac{1}{6}}=\dfrac{5}{6\sqrt[6]{x}}$

(3) $y=(x^2+4)^{\frac{1}{2}}$

$y'=\dfrac{1}{2}(x^2+4)^{-\frac{1}{2}}\cdot2x$

$=\dfrac{x}{\sqrt{x^2+4}}$

(4) $y=(3x+1)^{-\frac{1}{2}}$

$y'=-\dfrac{1}{2}(3x+1)^{-\frac{3}{2}}\times3$

$=-\dfrac{3}{2(3x+1)\sqrt{3x+1}}$

3 次の関数を微分せよ。

(1) $y=\dfrac{x^2-3x+2}{2x-3}$　　(2) $y=\dfrac{3}{(2x^2-1)^3}$　　(3) $y=\sqrt[3]{x^2+4x+5}$

(1) $y'=\dfrac{(2x-3)(2x-3)-(x^2-3x+2)\cdot2}{(2x-3)^2}$

$=\dfrac{4x^2-12x+9-2x^2+6x-4}{(2x-3)^2}=\dfrac{2x^2-6x+5}{(2x-3)^2}$

(2) $y=3(2x^2-1)^{-3}$

$y'=3\times(-3)(2x^2-1)^{-4}\cdot4x=\dfrac{-36x}{(2x^2-1)^4}$

(3) $y=(x^2+4x+5)^{\frac{1}{3}}$

$y'=\dfrac{1}{3}(x^2+4x+5)^{-\frac{2}{3}}\cdot(2x+4)$

$=\dfrac{2x+4}{3\sqrt[3]{(x^2+4x+5)^2}}$

4 次の関数を微分せよ。

(1) $y=\dfrac{x^3}{(5x+1)^2}$

(2) $y=\dfrac{\sqrt{1-x^2}}{1+x^2}$

(1) $y'=\dfrac{3x^2\cdot(5x+1)^2-x^3\cdot 2(5x+1)\times 5}{\{(5x+1)^2\}^2}=\dfrac{3x^2(5x+1)-5x^3\cdot 2}{(5x+1)^3}$

$=\dfrac{5x^3+3x^2}{(5x+1)^3}$

(2) $y'=\dfrac{(\sqrt{1-x^2})'\cdot(1+x^2)-\sqrt{1-x^2}\cdot 2x}{(1+x^2)^2}$

$=\dfrac{\dfrac{-x}{\sqrt{1-x^2}}(1+x^2)-\sqrt{1-x^2}\cdot 2x}{(1+x^2)^2}=\dfrac{x^3-3x}{(1+x^2)^2\sqrt{1-x^2}}$

5 次の関数を微分せよ。

(1) $y=(x-1)^2$

(2) $y=(3x-1)^3$

(3) $y=(2x-1)(x-2)^2$

(4) $y=(x^2+2x+3)^2$

(5) $y=\dfrac{1}{(2x^3+3)^2}$

(6) $y=\left(x+\dfrac{1}{x}\right)^3$

(1) $y'=2(x-1)$

(2) $y'=3(3x-1)^2\cdot 3$

$=9(3x-1)^2$

(3) $y'=2(x-2)^2+(2x-1)\cdot 2(x-2)\cdot 1$

$=(x-2)\{2(x-2)+2(2x-1)\}$

$=(x-2)(6x-6)=6(x-2)(x-1)$

(4) $y'=2(x^2+2x+3)\cdot(2x+2)$

$=4(x+1)(x^2+2x+3)$

(5) $y=(2x^3+3)^{-2}$

$y'=-2(2x^2+3)^{-3}\times 6x^2$

$=\dfrac{-12x^2}{(2x^2+3)^3}$

(6) $y'=3\left(x+\dfrac{1}{x}\right)^2\cdot\left(x+x^{-1}\right)'$

$=3\left(x+\dfrac{1}{x}\right)^2\cdot\left(1-x^{-2}\right)$

$=3\left(x+\dfrac{1}{x}\right)^2\left(1-\dfrac{1}{x^2}\right)$

6 次の関数を微分せよ。

(1) $y=(x^2+3x-5)^2$

(2) $y=\dfrac{1}{(x^2+x+1)^2}$

(3) $y=(x^2-3)^2(x+1)$

(4) $y=\sqrt[5]{x^3}$

(5) $y=\sqrt{9-x^2}$

(6) $y=\sqrt{\dfrac{x-1}{x+1}}$

(1) $y'=2(x^2+3x-5)\cdot(2x+3)$

(2) $y=(x^2+x+1)^{-2}$

$y'=-2(x^2+x+1)^{-3}\cdot(2x+1)$

$=\dfrac{-2(2x+1)}{(x^2+x+1)^3}$

(3) $y'=2(x^2-3)\cdot 2x(x+1)+(x^2-3)^2\cdot 1$

$=(x^2-3)\{4x(x+1)+x^2-3\}$

$=(x^2-3)(5x^2+4x-3)$

(4) $y=x^{\frac{3}{5}}$

$y'=\dfrac{3}{5}x^{-\frac{2}{5}}=\dfrac{3}{5\sqrt[5]{x^2}}$

(5) $y=(9-x^2)^{\frac{1}{2}}$

$y'=\dfrac{1}{2}(9-x^2)^{-\frac{1}{2}}\cdot(-2x)$

$=\dfrac{-x}{\sqrt{9-x^2}}$

(6) $y=\left(\dfrac{x-1}{x+1}\right)^{\frac{1}{2}}$

$y'=\dfrac{1}{2}\left(\dfrac{x-1}{x+1}\right)^{-\frac{1}{2}}\times\left(\dfrac{x-1}{x+1}\right)'$

$=\dfrac{1}{2}\sqrt{\dfrac{x+1}{x-1}}\cdot\dfrac{1\cdot(x+1)-(x-1)\cdot 1}{(x+1)^2}=\dfrac{1}{2}\sqrt{\dfrac{x+1}{x-1}}\times\dfrac{2}{(x+1)^2}$

$=\dfrac{1}{\sqrt{(x-1)(x+1)^3}}$