

6-6 対数関数の導関数

1 次の関数を微分せよ。

(1)  $y = \log 4x$

(2)  $y = \log|x-1|$

(3)  $y = \log_3(1-5x)$

(4)  $y = \log_{10}|3x-2|$

(5)  $y = \log|x^2-3|$

(6)  $y = \log\sqrt{x+1}$

(1)  $y' = \frac{1}{4x} \times 4$

(2)  $y' = \frac{1}{x-1} \cdot 1$

(3)  $y' = \frac{1 \cdot (-5)}{(1-5x) \log 3}$

$y' = \frac{1}{x}$

$y' = \frac{1}{x-1}$

$y' = \frac{-5}{(1-5x) \log 3}$

(4)  $y' = \frac{1 \cdot 3}{(3x-2) \log 10}$

(5)  $y' = \frac{2x}{x^2-3}$

$y' = \frac{3}{(3x-2) \log 10}$

(6)  $y' = \frac{\frac{1}{2} \cdot \frac{1}{\sqrt{x+1}}}{\sqrt{x+1}}$

(6)  $y = \frac{1}{2} \log(x+1)$

$y' = \frac{1}{2(x+1)}$

$= \frac{1}{2(x+1)}$

2 次の関数を微分せよ。

(1)  $y = x^2 \log x - \frac{1}{2}x^2$

(2)  $y = \log|\tan x|$

(3)  $y = (\log x)^3$

(1)  $y' = 2x \log x + x^2 \cdot \frac{1}{x} - \frac{1}{2} \cdot 2x$

(2)  $y' = \frac{1}{\tan x} \cdot (\tan x)'$

$y' = 2x \log x + x - x$

$= \frac{1}{\tan x} \cdot \frac{1}{\cos^2 x}$

$y' = 2x \log x$

$y' = \frac{1}{\sin x \cos x}$

(3)  $y' = 3(\log x)^2 \cdot \frac{1}{x}$

$y' = \frac{3(\log x)^2}{x}$

3 次の関数を微分せよ。

(1)  $y = \log |\log x|$       (2)  $y = \log \left| \frac{x-1}{x+1} \right|$       (3)  $y = \log_x a$

(4)  $y = \log \sqrt{\frac{x^2-1}{x^2+1}}$       (5)  $y = \log(x + \sqrt{x^2+4})$

(1)  $y' = \frac{1}{\log x} \cdot \frac{1}{x}$

$y' = \frac{1}{x \log x}$

(3)  $y = \log_x a$   
 $= \frac{\log a}{\log x}$

$y' = \log a \times \frac{-\frac{1}{x}}{(\log x)^2} = -\frac{\log a}{x(\log x)^2}$

(4)  $y = \frac{1}{2} \log \frac{x^2-1}{x^2+1}$

$= \frac{1}{2} \{ \log(x^2-1) - \log(x^2+1) \}$

$y' = \frac{1}{2} \left( \frac{2x}{x^2-1} - \frac{2x}{x^2+1} \right)$

$= \frac{2x}{x^2-1}$

(2)  $y' = \frac{1}{\frac{x-1}{x+1}} \times \left( \frac{x-1}{x+1} \right)'$

$= \frac{x+1}{x-1} \times \frac{x+1 - (x-1)}{(x+1)^2}$

$= \frac{2}{(x-1)(x+1)}$

(5)  $y' = \frac{1}{x + \sqrt{x^2+4}} \times (x + \sqrt{x^2+4})'$

$= \frac{1}{x + \sqrt{x^2+4}} \left( 1 + \frac{1}{2} \frac{1}{\sqrt{x^2+4}} \times 2x \right)$

$= \frac{1}{\sqrt{x^2+4}}$

4 次の関数を微分せよ。ただし、 $a$  は定数で、 $a > 0$ ,  $a \neq 1$  とする。

(1)  $y = \log(x^2+2)$       (2)  $y = \log \left| \frac{2x-1}{2x+1} \right|$       (3)  $y = \log |x^2-4|$

(4)  $y = \log(\sin x)$       (5)  $y = (\log x)^3$       (6)  $y = (x \log x - x)^2$

(7)  $y = \log_4 2x$       (8)  $y = \log_a(x^2-1)$

(1)  $y' = \frac{2x}{x^2+2}$

(2)  $y = \log |2x-1| - \log |2x+1|$

$y' = \frac{2}{2x-1} - \frac{2}{2x+1} = \frac{4}{4x^2-1}$

(3)  $y' = \frac{2x}{x^2-4}$

(5)  $y' = 3(\log x)^2 \cdot \frac{1}{x}$

(4)  $y' = \frac{\cos x}{\sin x}$

$y' = \frac{3(\log x)^2}{x}$

(6)  $y' = 2(x \log x - x) \cdot \left( 1 \cdot \log x + x \cdot \frac{1}{x} - 1 \right)$   
 $= 2(x \log x - x) \log x$

(7)  $y = \frac{\log 2x}{\log 4}$        $y' = \frac{\frac{2}{2x}}{\log 4} = \frac{1}{x \log 4} = \frac{1}{2x \log 2}$

(8)  $y' = \frac{2x}{(x^2-1) \log a}$