$\boxed{1}$ $(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \dfrac{1}{\cos^2 x}$ となることを示せ。

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left\{ \frac{\sin x(\cos h - 1)}{h} + \cos x \cdot \frac{\sin h}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{\sin x \cdot \sin^2 h}{h(\cos h + 1)} + \cos x \cdot \frac{\sin h}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \sin x \cdot \frac{\sin h}{h} \cdot \frac{\sin h}{\cos h + 1} + \cos x \cdot \frac{\sin h}{h} \right\}$$

$$= \sin x \cdot 1 \cdot \frac{0}{1+1} + \cos x \cdot 1$$

$$= \cos x$$

$$(\sin x)' = \cos x$$

$$\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left\{ \frac{\cos x(\cos h - 1)}{h} - \sin x \cdot \frac{\sin h}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{\cos x \cdot \sin^2 h}{h(\cos h + 1)} - \sin x \cdot \frac{\sin h}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \cos x \cdot \frac{\sin h}{h} \cdot \frac{\sin h}{\cos h + 1} - \sin x \cdot \frac{\sin h}{h} \right\}$$

$$= \cos x \cdot 1 \cdot \frac{0}{1+1} - \sin x \cdot 1$$

$$= -\sin x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x} \right)'$$

$$= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

2　次の関数を微分せよ。

(1)　$y=\sin x+\cos x$　　(2)　$y=\tan x+x$

(3)　$y=\cos 3x$　　(4)　$y=\sin\left(2x+\frac{\pi}{3}\right)$

(5)　$y=\tan 4x$　　(6)　$y=\cos x^2$

(7)　$y=\tan x^3$　　(8)　$y=\sin^3 x$

(9)　$y=\tan^4 x$　　(10)　$y=x^2\sin 3x^2$

(11)　$y=\sin x\cos^2 x$　　(12)　$y=\frac{x^2}{\cos x}$

(1) $y'=\cos x-\sin x$

(2) $y'=\frac{1}{\cos^2 x}+1$

(3) $y'=-\sin 3x\times 3$

$=-3\sin 3x$

(4) $y'=\cos\left(2x+\frac{\pi}{3}\right)\times 2$

$=2\cos\left(2x+\frac{\pi}{3}\right)$

(5) $y'=\frac{1}{\cos^2 4x}\times 4$

$y'=\frac{4}{\cos^2 4x}$

(6) $y'=-\sin x^2\times 2x$

$y'=-2x\sin x^2$

(7) $y'=\frac{1}{\cos^2 x^3}\times 3x^2$

$y'=\frac{3x^2}{\cos^2 x^3}$

(8) $y'=3\sin^2 x\times\cos x$

$y'=3\cos x\sin^2 x$

(9) $y'=4\tan^3 x\cdot\frac{1}{\cos^2 x}$

$y'=\frac{4\tan^3 x}{\cos^2 x}$

(10) $y'=2x\sin 3x^2+x^2\cos 3x^2\times 6x$

$y'=2x\sin 3x^2+6x^3\cos 3x^2$

(11) $y'=\cos x\cos^2 x+\sin x\cdot 2\cos x\cdot(-\sin x)$

$y'=\cos^3 x-2\sin^2 x\cos x$

(12) $y'=\frac{2x\cos x-x^2\cdot(-\sin x)}{\cos^2 x}$

$y'=\frac{2x\cos x+x^2\sin x}{\cos^2 x}$

3　次の関数を微分せよ。

$y=\cos^2 4x$

$y'=2\cos 4x\cdot(-\sin 4x)\times 4$

$y'=-8\cos 4x\sin 4x$

$=-4\cdot 2\cos 4x\sin 4x$

$=-4\sin 8x$

$\boxed{4}$ 次の関数を微分せよ。

(1) $y=\sin^2\left(2x+\dfrac{\pi}{6}\right)$　　(2) $y=\sin\sqrt{x^2-x+1}$

(3) $y=\sin^4 x\cos 4x$　　(4) $y=\sqrt{1+\cos^2 x}$

(5) $y=\dfrac{\cos x}{1-\sin x}$　　(6) $y=\left(\tan x+\dfrac{1}{\tan x}\right)^2$

(1) $y' = 2\sin\left(2x+\frac{\pi}{6}\right)\times\cos\left(2x+\frac{\pi}{6}\right)\times 2$

$y' = 4\sin\left(2x+\frac{\pi}{6}\right)\cos\left(2x+\frac{\pi}{6}\right) = 2\sin\left(4x+\frac{\pi}{3}\right)$

(2) $y' = \cos\sqrt{x^2-x+1}\times\frac{1}{2}\cdot\frac{1}{\sqrt{x^2-x+1}}\times(2x-1)$

$= \dfrac{(2x-1)\cos\sqrt{x^2-x+1}}{2\sqrt{x^2-x+1}}$

(3) $y' = 4\sin^3 x\times\cos x\cdot\cos 4x + \sin^4 x\cdot(-\sin 4x)\cdot 4$

$= 4\sin^3 x\cos x\cos 4x - 4\sin^4 x\sin 4x$

$= 4\sin^3 x(\cos x\cos 4x - \sin x\sin 4x)$

$= 4\sin^3 x\{\cos(x+4x)\}$

$= 4\sin^3 x\cos 5x$

(4) $y' = \frac{1}{2}\cdot\frac{1}{\sqrt{1+\cos^2 x}}\times 2\cos x\times(-\sin x)$

$= -\dfrac{2\sin x\cos x}{2\sqrt{1+\cos^2 x}} = -\dfrac{\sin 2x}{2\sqrt{1+\cos^2 x}}$

(5) $y' = \dfrac{-\sin x(1-\sin x)-\cos x(-\cos x)}{(1-\sin x)^2}$

$= \dfrac{-\sin x+\sin^2 x+\cos^2 x}{(1-\sin x)^2}$

$= \dfrac{1-\sin x}{(1-\sin x)^2} = \dfrac{1}{1-\sin x}$

(6) $y' = 2\left(\tan x+\frac{1}{\tan x}\right)\times\left(\frac{1}{\cos^2 x}-\frac{\frac{1}{\cos^2 x}}{\tan^2 x}\right)$

$= 2\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)\times\left(\frac{1}{\cos^2 x}-\frac{\cos^2 x}{\sin^2 x}\times\frac{1}{\cos^2 x}\right)$

$= 2\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right)$

$= 2\cdot\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x}\times\dfrac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}$

$= 2\cdot\dfrac{-\cos 2x}{\sin^3 x\cos^3 x} = 2\cdot 8\cdot\dfrac{-\cos 2x}{(2\sin x\cos x)^3} = -\dfrac{16\cos 2x}{\sin^3 2x}$