

6-2 積と商の導関数

1 次の関数を微分せよ。

(1) $y = 2x^5 + 3x^4 - 5x^2 + 1$

(3) $y = (x^2 + 1)(x^2 - x - 3)$

(1) $y' = 2 \cdot 5x^4 + 3 \cdot 4x^3 - 5 \cdot 2x$
 $y' = 10x^4 + 12x^3 - 10x$

(2) $y = -3x^7 + 5x^3 - 4x$

(4) $y = (x^3 + x)(x^2 - 2)$

(2) $y' = -3 \cdot 7x^6 + 5 \cdot 3x^2 - 4$
 $y' = -21x^6 + 15x^2 - 4$

(3) $y' = 2x \cdot (x^2 - x - 3) + (x^2 + 1)(2x - 1)$
 $= 2x^3 - 2x^2 - 6x + 2x^3 - x^2 + 2x - 1$
 $= 4x^3 - 3x^2 - 4x - 1$

2 次の関数を微分せよ。

(1) $y = (x-2)^2(x-3)^3$

(2) $y = (x+2)(x-1)(x-5)$

(1) $y' = \{(x-2)^2\}'(x-3)^3 + (x-2)^2 \{(x-3)^3\}'$
 $= 2(x-2)(x-3)^3 + (x-2)^2 \cdot 3(x-3)^2$
 $= (x-2)(x-3)^2 \{2(x-3) + 3(x-2)\}$
 $= (x-2)(x-3)^2 (5x-12)$

(2) $y' = (x+2)'(x-1)(x-5) + (x+2) \{(x-1)(x-5)\}'$
 $= (x+2) \{(x-1)'(x-5) + (x-1)(x-5)'\}$
 $= (x+2) \{1(x-5) + (x-1)(x-5)\}$
 $= (x+2) \{x-5 + (x-1)(x-5)\}$
 $= (x+2) \{x-5 + x^2 - 6x + 5\}$
 $= (x+2)(x^2 - 5x)$
 $= x^3 - 5x^2 + 2x^2 - 10x$
 $= x^3 - 3x^2 - 10x$

3 次の関数を微分せよ。

(1) $y = \frac{1}{3x-1}$

(2) $y = \frac{x}{x^2+1}$

(3) $y = \frac{x-2}{x^2+x+1}$

(4) $y = \frac{x^2-3x+2}{2x-3}$

(1) $y' = -\frac{(3x-1)'}{(3x-1)^2} = -\frac{3}{(3x-1)^2}$

(2) $y' = \frac{x'(x^2+1) - x(x^2+1)'}{(x^2+1)^2} = \frac{x^2+1 - x(2x)}{(x^2+1)^2}$
 $= \frac{x^2+1 - 2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$

(3) $y' = \frac{1 \cdot (x^2+x+1)' - (x-2)(2x+1)'}{(x^2+x+1)^2} = \frac{x^2+x+1 - (2x^2-3x-2)}{(x^2+x+1)^2}$
 $= \frac{-x^2+4x+3}{(x^2+x+1)^2}$

(4) $y' = \frac{(2x-3)(2x-3) - (x^2-3x+2) \cdot 2}{(2x-3)^2} = \frac{4x^2 - 12x + 9 - 2x^2 + 6x - 4}{(2x-3)^2}$
 $= \frac{2x^2 - 6x + 5}{(2x-3)^2}$

6-2 積と商の導関数

4 次の関数を微分せよ。

(1) $y = \frac{1}{x+1}$

(2) $y = \frac{2x}{x+3}$

(3) $y = \frac{1}{x^2-1}$

(4) $y = \frac{x-1}{x^2+1}$

(5) $y = \frac{x}{x^2-x+1}$

(6) $y = \frac{x^3-4x+1}{x-2}$

(1) $y' = -\frac{1}{(x+1)^2}$

(2) $y' = \frac{2(x+3) - 2x - 1}{(x+3)^2} = \frac{6}{(x+3)^2}$

(3) $y' = -\frac{2x}{(x^2-1)^2}$

(4) $y' = \frac{1 \cdot (x^2+1) - (x-1) \cdot 2x}{(x^2+1)^2} = \frac{-x^2+2x+1}{(x+1)^2}$

(5) $y' = \frac{1 \cdot (x^2-x+1) - x(2x-1)}{(x^2-x+1)^2} = \frac{-x^2+1}{(x^2-x+1)^2}$

(6) $y' = \frac{(3x^2-4)(x-2) - (x^3-4x+1) \cdot 1}{(x-2)^2} = \frac{2x^3-6x^2+7}{(x-2)^2}$

5 次の関数を微分せよ。

(1) $y = 2x^4 + 3x^3 + 4x^2 - 5$

(2) $y = (x^2+3x)(x^2-2)$

(3) $y = (x^2-2x-3)(x^2+4)$

(4) $y = (x-1)(x+2)(x-3)$

(5) $y = \frac{1}{x^4}$

(6) $y = \frac{x^2+3x-2}{x}$

(7) $y = \frac{x-2}{x^2}$

(8) $y = \frac{1}{x^2-3x+2}$

(9) $y = \frac{x+1}{x-1}$

(10) $y = \frac{x^2+2x-2}{x^2+1}$

(1) $y' = 8x^3 + 9x^2 + 8x$

(2) $y' = (2x+3)(x^2-2) + (x^2+3x) \cdot 2x = 2x^3-4x+3x^2-6 + 2x^3+6x^2 = 4x^3+9x^2-4x-6$

(3) $y' = (2x-2)(x^2+4) + (x^2-2x-3) \cdot 2x = 2x^3+8x-2x^2-8 + 2x^3-4x^2-6x = 4x^3-6x^2+2x-8$

(4) $y' = 1 \cdot (x+2)(x-3) + (x-1) \{ 1 \cdot (x-3) + (x+2) \cdot 1 \} = x^2-x-6 + (x-1)(2x-1) = x^2-x-6 + 2x^2-3x+1 = 3x^2-4x-5$

(5) $y' = \frac{-4x^3}{(x^4)^2} = -\frac{4x^3}{x^8} = -\frac{4}{x^5}$

(6) $y = x+3 - \frac{2}{x}, y' = 1 - \frac{-2}{x^2} = 1 + \frac{2}{x^2}$

(7) $y' = \frac{1 \cdot x^2 - (x-2) \cdot 2x}{(x^2)^2} = \frac{x^2 - 2x^2 + 4x}{x^4} = \frac{-x^2 + 4x}{x^4} = \frac{-x+4}{x^3}$

(8) $y' = \frac{-(2x-3)}{(x^2-3x+2)^2}$ (9) $y' = \frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$

(10) $y' = \frac{(2x+2)(x^2+1) - (x^2+2x-2) \cdot 2x}{(x^2+1)^2} = \frac{-2x^2+6x+2}{(x^2+1)^2}$