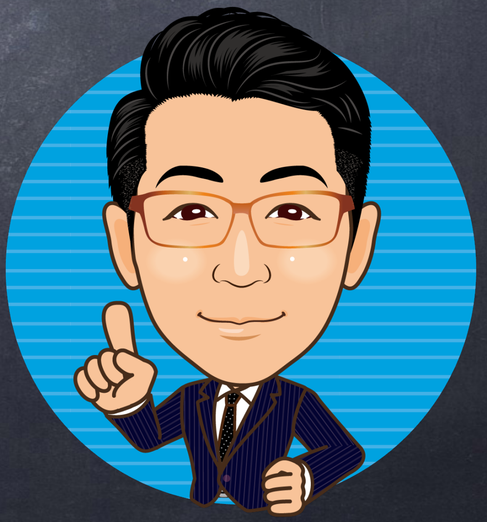
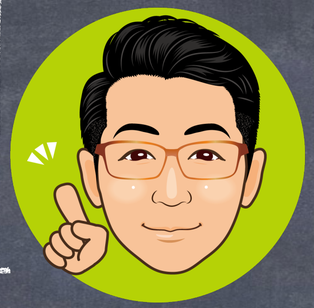
テーマ：
グラフの書き方①



(例)

$y = e^{-\frac{x^2}{2}}$ のグラフ



増減，凹凸，漸近線

$y' = 0$ のとき $x = 0$

$y'' = 0$ のとき $x = \pm 1$

$$y' = e^{-\frac{x^2}{2}} \cdot (-x) = -xe^{-\frac{x^2}{2}}$$

$$y'' = -e^{-\frac{x^2}{2}} + (-x) \cdot \left(-xe^{-\frac{x^2}{2}}\right)$$

$$= (x^2 - 1)e^{-\frac{x^2}{2}}$$

x	$\cdots$	-1	$\cdots$	0	$\cdots$	1	$\cdots$
y'	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$+$	0	$-$	$-$	$-$	0	$+$
y	↗	変曲点	↗	極大	↘	変曲点	↘

⇐ ⇒

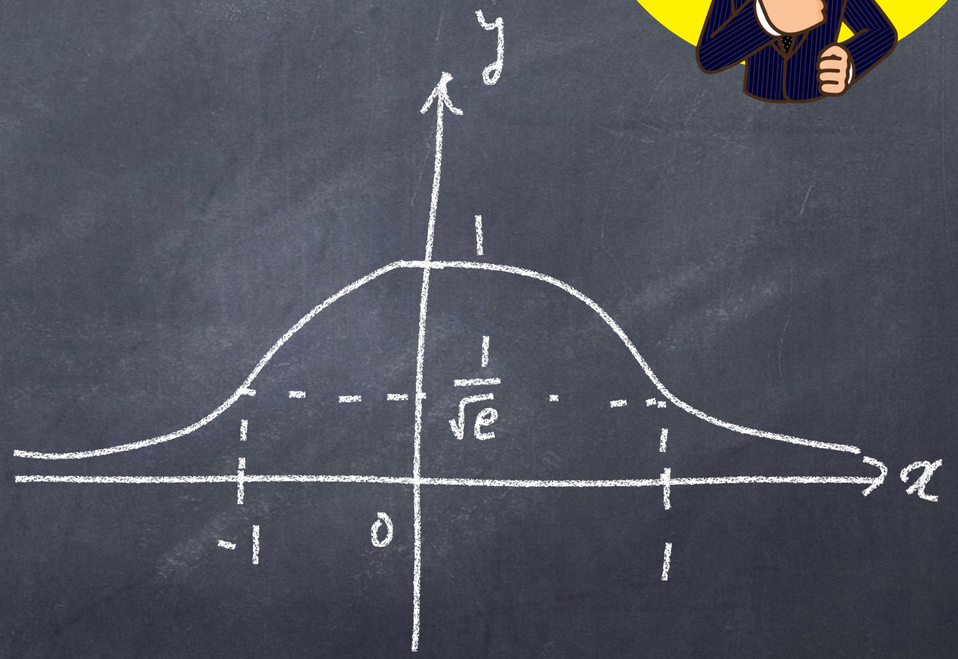
(例)

$y = e^{-\frac{x^2}{2}}$ のグラフ

x	$\cdots$	-1	$\cdots$	0	$\cdots$	1	$\cdots$
y'	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$+$	0	$-$	$-$	$-$	0	$+$
y	↗	$\frac{1}{\sqrt{e}}$	∩	1	↘	$\frac{1}{\sqrt{e}}$	∪

$\lim_{x \to \infty} e^{-\frac{x^2}{2}} = 0$

$\lim_{x \to -\infty} e^{-\frac{x^2}{2}} = 0$



漸近線は x 軸 $(y=0)$

(例)

$y = \dfrac{x^2}{x-1}$ の y'' を求めよ

$y = \dfrac{x^2}{x-1}$ は $x \neq 1$

$$\begin{array}{r} x+1 \\ x-1 \overline{)\, x^2} \\ x^2 - x \\ \hline x \\ x-1 \\ \hline 1 \end{array}$$

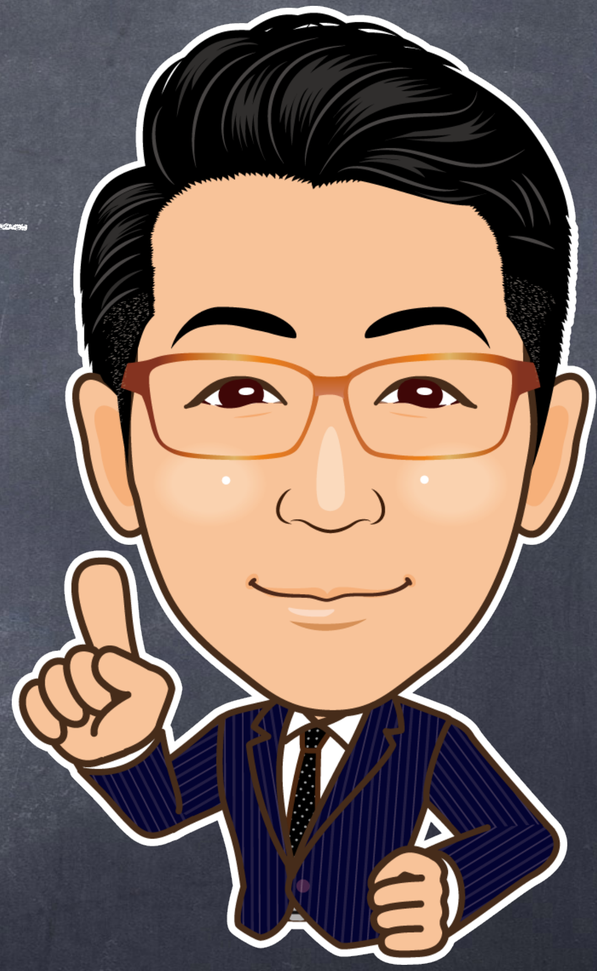
$$y = \frac{x^2}{x-1} = x+1+\frac{1}{x-1}$$

$$y' = 1 - \frac{1}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

$$y'' = \frac{2}{(x-1)^3}$$

$y' = 0$ のとき $x = 0, 2$

$x > 1$ のとき $y'' > 0$, $x < 1$ のとき $y'' < 0$



(ex)

$$y = \frac{x^2}{x-1}$$ の y'' まで

$$y = x + 1 + \frac{1}{x-1}$$

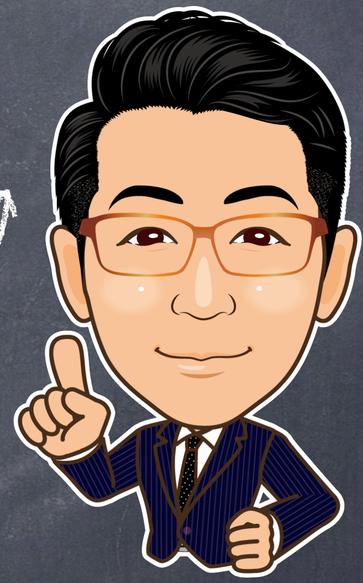
$$y' = \frac{x(x-2)}{(x-1)^2}$$

$$y'' = \frac{2}{(x-1)^3}$$

x	$\cdots$	0	$\cdots$	1	$\cdots$	2	$\cdots$
y'	$+$	0	$-$	/	$-$	0	$+$
y''	$-$	$-$	$-$	/	$+$	$+$	$+$
y	↗		↘	/	↘		↗

$\lim_{x \to 1+0} y = \infty$, $\lim_{x \to 1-0} y = -\infty$

$\lim_{x \to \infty} y = \infty$, $\lim_{x \to -\infty} y = -\infty$

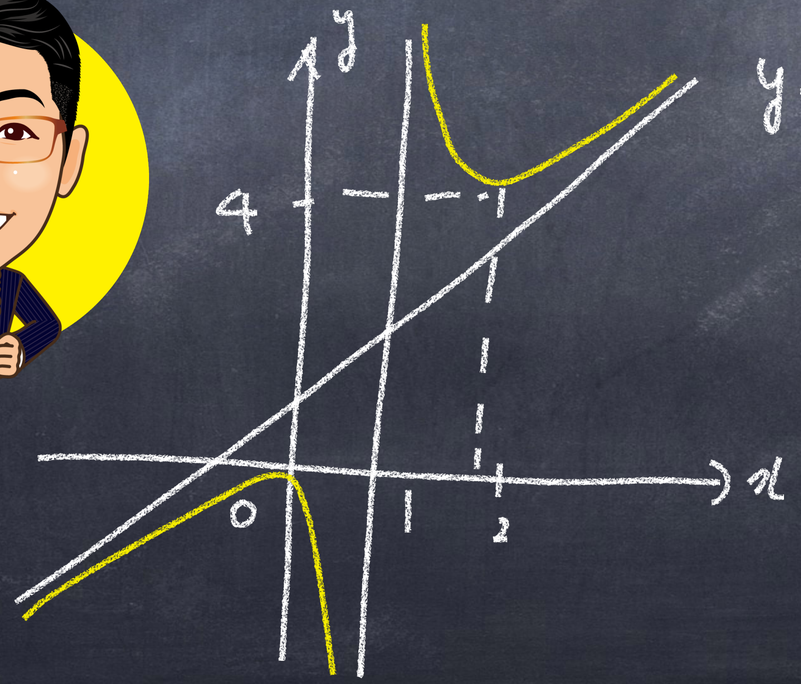


(例)

$$y = \frac{x^2}{x-1}$$ の y'' を求めよ

x	…	0	…	1	…	2	…
y'	+	0	−	/	−	0	+
y''	−	−	−	/	+	+	+
y	↗	0	↘	/	↘	4	↗

$$y = x + 1 + \frac{1}{x-1}$$



$$\lim_{x \to \infty} \{y - (x+1)\} = 0$$

$$\lim_{x \to -\infty} \{y - (x+1)\} = 0$$

$y = x + 1$ は漸近線

直線 $x = 1$